

A Puzzle about Random Connection

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Given an undirected graph containing n ($n > 2$) nodes, the probability that arbitrary pairs of nodes are connected by an edge is p . Given two nodes randomly, what is the probability that there exists a path between them?

Lemma 1. *For any undirected graph which contains m nodes and the probability that arbitrary pair of nodes are connected by an edge is p , the probability that the graph is connected is*

$$f(m) = 1 - \sum_{i=1}^{m-1} C_{m-1}^{i-1} f(i) (1-p)^{i(n-i)}. \quad (1)$$

Proof. The graph may have many connected components. For arbitrary node v , the probability that the size of components that v belongs to is i is

$$g(i) = C_{n-1}^{i-1} f(i) (1-p)^{i(n-i)}. \quad (2)$$

We can get the proof based on $\sum_{i=1}^n g(i) = 1$. □

There exists C_i^2 paths in a connected components, so the answer is

$$\frac{1}{C_n^2} \sum_{i=1}^n C_i^2 f(i). \quad (3)$$